Factoring Polynomials
GCF, grouping, two terms

Factoring means to write as a product and is used to simplify expressions or solve equations. The first step in factoring always begins by checking if there is a greatest common factor.

Example: Multiply the term using distribution.
- ❖ $2x(3x^2 - 5) = 6x^3 - 10x$

"Think about working the problem in reverse, in other words factor out the GCF."

Example: Factor
- ❖ $6x^3 - 10x$ The greatest term that divides into both is 2x.
- ❖ $2x\left(\dfrac{6x^3}{2x} - \dfrac{10x}{2x}\right)$ Write 2x on the outside and divide each term by 2x

    Reduce inside parenthesis and the answer becomes: $2x(3x^2 - 5)$

➢ How to factor depends on the number of term in the polynomial.

4terms   Factoring four terms by **GROUPING**

- ❖ $3x^4 + 3x^3 - 21x^2 - 21x$ ⇐ group the first two and last two terms then factor each

- ❖ $3x^3(x+1) - 7x(x+1)$ ⇐ factor out the matching factor $(x+1)$ and write what is left

- ❖ $(x+1)(3x^3 - 7)$ ⇐ include parentheses around each factor. Done ☺

2terms   Factoring two terms by **FORMULA**

Does the polynomial have two terms?

a) Is it difference of squares? $a^2 - b^2 = (a+b)(a-b)$

b) Is it the sum of squares? $a^2 + b^2 = $ prime (not factorable)

c) Is it the difference of cubes? $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

d) Is it the sum of cubes? $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

Example: Factor $x^2 - 9$ two terms must use difference of squares

- ❖ $x^2 - 9 \Rightarrow x^2 - 3^2 \Rightarrow (x+3)(x-3)$

Example: Factor $w^3 - 8$ two terms must use difference of cubes

❖ $w^3 - 8 \Rightarrow w^3 - 2^3 \Rightarrow$ So, a = w and b = 2 now insert into the equation.

❖ $(w - 2)(w^2 + 2w + 2^2)$ the answer is $\rightarrow (w - 2)(w^2 + 2w + 4)$

Practice Problems

Factor each polynomial completely.

1) $14 - 7y$

2) $x^3 + 1000$

3) $10a^2 - 20a$

4) $49y^2 - 25x^2$

5) $8w^3 - 125$

6) $25x^2 - 16$

7) $m(m + 2n) + n(m + 2n)$

8) $m^3 + 2m^2 + 5m + 10$

9) $a^2b - 4a - ab^4 + 4b^3$

10) $y^2 - 3y - xy + 3x$

11) $10t^3 - 2t^2s^2 - 5ts + s^3$

12) $12xy^3 + 27x^3y$

Answer Key

1) $-7(y - 2)$

2) $(x + 10)(x^2 - 10x + 100)$

3) $10a(a - 2)$

4) $(7y + 5x)(7y - 5x)$

5) $(2w - 5)(4w^2 + 10w + 25)$

6) $(5x + 4)(5x - 4)$

7) $(m + 2n)(m + n)$

8) $(m + 2)(m^2 + 5)$

9) $(ab - 4)(a - b^3)$

10) $(y - 3)(y - x)$

11) $(2t^2 - s^2)(2t^2 - 5)$

12) $3xy(4y^2 + 9x^2)$